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Extended wave–particle description of longitudinal photons

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Abstract. On the basis of the two-wave particle model, the nonlinear wave hypothesis and the tachyonic theory of elementary particle structure, the propagation of electromagnetic fields in phase-locked cavities and waveguides has been investigated. It is shown that in a pull-phase-locked cavity, a Doppler-shifted radiation may be considered as a nonlinear system of time-like and space-like waves which *lock* to form the plane photon-like wave. In the particle picture, it may be interpreted as photon conversion into a system composed of both bradyonic and tachyonic components which trap each other in a relativistically invariant way. The considerations are extended to include the case of radiation transmitted inside waveguides. The relativistic mechanics and field description of longitudinal massive photons associated with trapped radiation have been developed. The time-like Proca wave equation and its space-like counterpart are derived.

1. Introduction

The construction of classical, semi-classical and quantum models of a massive particle has had a long and distinguished history. Recent research in this field has brought forth a few significant results which permit the construction of new wave models of a particle and better understanding of its internal structure. One of the most interesting concepts is probably the one by Jennison and Drinkwater (1977) and Jennison (1978, 1980, 1983, 1986), who investigated the particle as a relativistic phase-locked cavity with an internal standing electromagnetic wave. They have shown that such trapped radiation has inertial properties of a particle, and, on the other hand, all particles having an inertial mass may be considered as consisting of trapped radiation. So, intrinsically stable objects can be formed from a few precise frequencies of radiation (Jennison and Drinkwater 1977).

Another interesting model, the so-called *two-wave model*, has been developed by Horodecki (1982, 1983, 1984, 1988a,b), Das (1984, 1986, 1988) and Elbaz (1983, 1985, 1986, 1987, 1988), in the framework of the de Broglie postulate of wave–particle duality and the *many-wave* hypothesis (Horodecki 1982, Kostro 1985). In the two-wave description of matter, it is assumed that a massive particle can be described in terms of the time-like component characterized by the time-like 4-momentum $p^{\mu} \equiv (E/c, \boldsymbol{p})$ associated with de Broglie wave (B-wave), and the space-like component described by the space-like 4-momentum $p'^{\mu} \equiv (E'/c, \boldsymbol{p}')$ related to transformed Compton wave (D-wave). In this connection, the matter waves of second kind (D-waves) have a tachyon-like characteristic, e.g. they cannot be localized in space and their group velocity is greater than the light velocity in vacuum (Horodecki 1988a). The two-wave

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model implies the possibility of a space-like correlation inside quantum systems, and represents the space-like *face* of a particle.

In this work we propose to consider the Jennison–Drinkwater concept in the framework of (i) the two-wave particle model, (ii) the nonlinear wave hypothesis (Horodecki 1983, 1984, 1988a,b) and (iii) the tachyonic theory of elementary particle structure (Corben 1977, 1978a,b, Castorina and Recami 1978, Rosen and Szamosi 1980, Recami 1986). We shall also be concerned with a generalization of Jennison–Drinkwater results over the case of radiation trapped in waveguides (which are nothing else but two-dimensional phase-locked cavities with the third dimension *open*), as well as with the formulation of the relativistic mechanics and the field description of associated longitudinal photons.

2. General formulation

Let us recall first the most important concepts relevant in the framework of our consideration. As already stated, in the two-wave model, a particle with a rest mass m_0 may be complementarily characterized by the time-like and the space-like 4-momenta satisfying the equations (Das 1988, Horodecki 1988a,b)

$$p_\mu p^\mu = m_0^2 c^2 \qquad p^\mu = m_0 v^\mu \qquad v^\mu = (\gamma c, \gamma \beta c) \tag{1a,b,c}$$

$$p'_\mu p'^\mu = -m_0^2 c^2 \qquad p'^\mu = m_0 v'^\mu \qquad v'^\mu = (\gamma \beta c, \gamma c) \tag{2a,b,c}$$

where $\gamma = (1-\beta^2)^{-1/2}$, c/c is the unit vector along the direction of the 3-velocity of the particle, and c is the light velocity. A look into (1) and (2) reveals that components of the 4-momenta can be expressed in the following formulae

$$p_0 = mc = m'\beta' c \qquad \boldsymbol{p} = m\boldsymbol{v} = m'\boldsymbol{c} \qquad m' = m_0(\beta'^2 - 1)^{-1/2} \tag{3a,b,c}$$

$$p'_0 = mv = m'c \qquad \boldsymbol{p}' = m\boldsymbol{c} = m'\beta'\boldsymbol{c} \qquad \beta\beta' = 1. \tag{4a,b,c}$$

Hence, the particle dynamics is described by an invariant interaction condition

$$p_\mu p'^\mu = 0. \tag{5}$$

The double wave–particle description is given by the set of correspondences (Das 1988, Horodecki 1988a)

$$p^\mu = \hbar k^\mu \qquad p'^\mu = \hbar k'^\mu \tag{6a,b}$$

$$V_{\mathrm{G}} = V'_{\mathrm{P}} = \beta c < c \qquad V'_{\mathrm{G}} = V_{\mathrm{P}} = \beta' c > c \tag{7}$$

where V_{G} (V_{P}) is a group (phase) velocity of matter waves, and $k^\mu \equiv (\omega/c, \boldsymbol{k})$, $k'^\mu \equiv (\omega'/c, \boldsymbol{k}')$ are the wave 4-vectors of the B- and D-waves respectively. The field equations for the time-like and the space-like components are given by the formulae (Elbaz 1985, 1986)

$$(\partial_\mu \partial^\mu + m^2)\psi = 0 \qquad \partial_\mu \psi \partial^\mu \psi' = 0 \qquad (\partial_\mu \partial^\mu - m^2)\psi' = 0 \tag{8a,b,c}$$

where $m = m_0 c/\hbar$, and we use the metric with signature $(+---)$.

In accordance with the nonlinear wave hypothesis, a massive particle in motion represents a nonlinear non-dispersive wave packet (C-wave) which involves an internal spectrum of matter waves governed by a nonlinear propagation law (Horodecki 1983, 1984). In particular, it is assumed that de Broglie oscillations of a frequency ω excite in a vacuum medium oscillations of a low frequency ω' and the Doppler shifted frequencies $\omega - \omega'$ and $\omega + \omega'$ arise (Horodecki 1988a)

$$\omega_0^2 = (\omega - \omega')(\omega + \omega') \tag{9}$$

where ω_0 is the Compton frequency connected with a particle at rest. The quantities ω and ω' are interpreted as the harmonics of the internal spectrum of the de Broglie extended particle being a nonlinear C-wave excitation of the vacuum field (Horodecki 1988a).

Recently, a trend has been developed to suppose that tachyons (space-like, faster-than-light objects) may play a role in elementary particle structure (Recami 1968, 1969, Hamamoto 1974, Akiba 1976, Rafanelli 1976, 1978, Van der Spuy 1978, Rosen and Szamosi 1980). In particular it was shown (Corben 1977, 1978a,b, Castorina and Recami 1978) that a free bradyon (time-like, slower-than-light object) with a rest mass m_0 and a free tachyon with a rest mass m_0' can trap each other in a relativistically invariant way. If $m_0 > m_0'$ the compound particle is always a bradyon with a rest mass $M_0 = (m_0^2 - m_0'^2)^{1/2}$, described by the wavefunction $\Psi = \psi\psi'$ satisfying the wave equation $\partial_\mu \partial^\mu \Psi = -(M_0 c/\hbar)^2 \Psi$, with respect to the invariant interaction condition $\partial_\mu \psi \partial^\mu \psi' = 0$. Such a bradyon–tachyon compound may be considered as a coupled pair of particles associated with the time-like ψ wave and the space-like ψ' one, which interact and *lock* to form the plane wave Ψ, that is time-like when $m_0 > m_0'$ (Recami 1986 p112).

3. Massless photons and ponderable matter

Taking the formulae (9), (6*a*,*b*), (3*a*) and (4*a*) into account, one obtains

$$m_0^2 = (m - m')(m + m') \qquad m' = \beta m \tag{10a,b}$$

which relates masses of the time-like and the space-like components of a particle. Equation (10*a*) is equivalent to (9), and clearly shows that m and m' may be termed as the internal mass spectrum of a particle, or in other words, that a massive particle in motion can be regarded as a composite object with both bradyonic and tachyonic components. Analysing (10*a*,*b*), it is clearly seen that at the luminal velocity, $\beta = 1$, $m' = m$, $m_0 = 0$, hence, *luxons* (e.g. luminal particles of photon, neutrino or graviton type) appear as massless objects. The above case features a great resemblance with conversion of particle–antiparticle pairs (both of the same mass) into photons, accompanied by the disappearance of rest mass and photons creation. According to the two-wave model, in the reverse phenomenon (for example, Delbrück scattering), besides the *ordinary* time-like component also the space-like one should be created.

It must be emphasized here, that the appearance of the tachyonic components and the associated superluminal velocity should not be regarded as an inconsistency of the concept under consideration. There are well-known Salam–Weinberg type models for which the gauge symmetry is spontaneously broken by filling the vacuum with tachyons. In this case such tachyons are the Higgs-field particles which can be considered formally as tachyons converted into bradyons (Recami 1986, p117). The created tachyonic field may be interpreted as a component of a *virtual* massive photon (or Higgs-like particle), whose 4-momentum is orthogonal to the 4-momentum of the bradyonic component (Horodecki 1988a). It is worth noting moreover that such authors as Barut (1969) and Barut and Nagel (1977) have shown that the space-like particle states may play an important role in the second-order processes such as the Compton effect, the electromagnetic polarizabilities and the Delbrück scattering.

For our purpose, the most important conclusion which can be drawn from the above consideration is that in the process of conversion of massless photons into ponderable matter not only the *ordinary* time-like component should be created, but also the space-like one associated with the tachyonic field referred to as a matter wave of second kind (D-wave).

The photon characteristic and its conjugated wave characteristic may be given approximately by the 4-momentum and the wave 4-vector $P^\mu \equiv (E_p/c, \boldsymbol{p}_p)$, $K^\mu \equiv (\omega_p/c, \boldsymbol{k}_p)$, whereas the luminal wave–particle duality is represented by

$$K^\mu = \hbar P^\mu \qquad V_{\mathrm{G}} = V_{\mathrm{P}} = c. \tag{11a,b}$$

The free photon and the beam of electromagnetic rays are exactly massless; however, in the case of cloud radiation the situation is quite different. Let us consider a system of N identical photons moving along different space directions, characterized by the total 4-momentum

$$P_s^\mu = \left(N\hbar\omega_p/c, \sum_{i=1}^{N} \boldsymbol{n}_i \hbar\omega_p/c\right) \tag{12a}$$

where $\boldsymbol{n}_i$ is the unit vector along the ith photon motion. It is easy to see that the photon system as a whole has a non-zero rest mass

$$M_0 = \hbar\omega_p c^{-2}\left(N^2 - \sum_{i,j=1}^{N} \cos\theta_{ij}\right)^{1/2} \qquad \cos\theta_{ij} = \boldsymbol{n}_i \boldsymbol{n}_j \tag{12b}$$

in the case when

$$\theta_{ij} = \cos^{-1}(\boldsymbol{n}_i \boldsymbol{n}_j) \neq 0 \qquad i \neq j. \tag{13}$$

Such a cloud of electromagnetic radiation composed of massless photons is endowed with a non-zero rest mass, and may generate its own gravitational field, as well as may be affected by external gravitational fields. For the two-photon system, equation (12*b*) reduces to $M_0 = \hbar\omega_p c^{-2}(2 - 2\cos\theta)^{1/2}$, which indicates that a non-zero rest mass $2\hbar\omega_p/c^2$ may be attributed to the associated standing electromagnetic wave ($\theta = \pi$). This is a key to understanding the result of Jennison and Drinkwater, who proved that standing electomagnetic waves trapped in a phase-locked cavity have inertial properties of ponderable matter, and may be employed as a wave model of a massive particle at rest. We find here some analogy with the ideas of Elbaz (1985, 1986, 1987, 1988) and Kostro (1985).

4. Photon conversion into bradyon–tachyon pairs

Let us consider a phase-locked cavity filled with the standing electromagnetic waves, such as in the papers of Jennison and Drinkwater (1977) and Jennison (1978). When the motive reflector element on the wall of the cavity, is moved forward at a velocity v_1 relative to the laboratory, the internal frequency ω_0 is Doppler shifted according to the formula

$$\omega_1 = \omega_0 \left(\frac{1 + v_1/c}{1 - v_1/c}\right)^{1/2} . \tag{14}$$

The internal frequency ω_0 is received at the motive reflecting element and at a frequency ω_1, whereas, in the laboratory system it is reflected at the frequency

$$\omega = \omega_1 \left(\frac{1 + v_1/c}{1 - v_1/c}\right)^{1/2} = \omega_0 \left(\frac{1 + v_1/c}{1 - v_1/c}\right) = \omega_0 \left(\frac{1 + v/c}{1 - v/c}\right)^{1/2} \tag{15}$$

where $v = 2v_1/(1 + v_1^2/c^2)$ is the velocity of the following mirror element (placed on the second wall of the cavity) moving at a velocity v, such that the frequency ω is restored to the original value ω_0 (Jennison and Drinkwater 1977, Jennison 1978). If we restrict our consideration to one-dimensional motion along the z-axis, the radiation inside the phase-locked cavity propagates relative to the laboratory system with the Doppler-shifted frequency, in accordance with the Maxwell equation

$$\partial_\mu \partial^\mu \Psi = 0 \qquad \Psi = \exp\left[\mathrm{i}\omega_0 c^{-1} \left(\frac{1 + v/c}{1 - v/c}\right)^{1/2} (\boldsymbol{n}_z \boldsymbol{r} - ct)\right] \tag{16a,b}$$

where $\boldsymbol{n}_z$ is the unit vector along the direction of 3-velocity of the following mirror, which coincides with the direction of photon propagation. The solution (16*b*) may alternatively be written as follows:

$$\begin{aligned}\Psi &= \exp[\mathrm{i}\omega_0 c^{-1} \gamma(1 + \beta)(\boldsymbol{n}_z \boldsymbol{r} - ct)] \\ &= \exp[\mathrm{i}\omega_0 c^{-1} \gamma(\beta \boldsymbol{n}_z \boldsymbol{r} - ct)] \exp[\mathrm{i}\omega_0 c^{-1} \gamma(\boldsymbol{n}_z \boldsymbol{r} - \beta ct)]\end{aligned} \tag{17}$$

which by using the result of the previous section $m_0 = \omega_0 \hbar/c^2$, and relations (1), (2), turns out to be

$$\begin{aligned}\Psi &= \exp[\mathrm{i}m\gamma(\beta \boldsymbol{n}_z \boldsymbol{r} - ct)] \exp[\mathrm{i}m\gamma(\boldsymbol{n}_z \boldsymbol{r} - \beta ct)] \\ &= \exp[\mathrm{i}\hbar^{-1}(\boldsymbol{p}_z \boldsymbol{r} - Et)] \exp[\mathrm{i}\hbar^{-1}(\boldsymbol{p}'_z \boldsymbol{r} - E't)] = \psi\psi' .\end{aligned} \tag{18}$$

Substitution of (18) into (16*a*) leads to

$$\partial_\mu \partial^\mu \psi\psi' = \psi' \partial_\mu \partial^\mu \psi + \psi \partial_\mu \partial^\mu \psi' + 2\partial_\mu \psi \partial^\mu \psi' = 0 \tag{19}$$

which by taking advantage of the condition (8*b*) yields

$$\partial_\mu \partial^\mu \psi/\psi + \partial_\mu \partial^\mu \psi'/\psi' = 0. \tag{20}$$

By differentiating ψ and ψ' twice with respect to x^μ coordinates, and bearing in mind (1*a*) and (2*a*), we get

$$\partial_\mu \partial^\mu \psi = -m^2 \psi \qquad \partial_\mu \partial^\mu \psi' = m^2 \psi' \tag{21}$$

which are nothing but the field equations (8*a*) and (8*c*), for the time-like and the space-like fields. Moreover, equations (18–21) are consistent with the tachyonic theory of elementary particle structure, and may be viewed as a special case of the Corben (1977, 1978a,b) and Castorina and Recami (1978) results for the bradyonic–tachyonic components endowed with the same rest mass. Now, it is apparent that the radiation under motion of the motive reflector wall is Doppler shifted, and transforms into a nonlinear wave system composed of the B- and D-waves which *lock* to the form of luminal wave. In the corpuscular interpretation, photons in a phase-locked cavity undergo a conversion into bradyon–tachyon components coupled to each other in the relativistically invariant way. The compound particle has photon-like characteristic, e.g. it moves at the light velocity and has zero rest mass.

Recently, a trend has been developed (Elbaz 1985, 1986, 1987, 1988, Kostro 1985) to suppose that a massive particle at rest is associated with an intrinsic standing photon-like wave, characterized by the wave 4-vector $K^\mu = (\omega_0/c, n\omega_0/c)$, where $\omega_0 = m_0 c^2/\hbar$ is a Compton frequency. In view of the outlined theory it is clear, because a standing electromagnetic wave trapped into phase-locked cavity, for $v = 0$, represents a massive particle at rest.

5. Relativistic mechanics of longitudinal photons

Free photons are *par excellence* relativistic particles moving in vacuum at the light velocity, and characterized by the zero rest mass and the spin $J = 1$. The electromagnetic vector fields associated with such photons (termed the *transverse* photons) are perpendicular to the wave 3-vector determining the direction of wave propagation. The transverse photons associated with TEM waves are exactly massless; the extraterrestial limit on the photon mass obtained by Goldhaber and Nieto is $m_0 \leqslant 4\times10^{-48}$ g, and the Williams–Faller–Hill laboratory experiment led to a slightly different result $m_0 \leqslant 1.6 \times 10^{-47}$ g. On the contrary, photons which have *longitudinal* components of electromagnetic fields, cannot be massless (Bass and Schrödinger 1955, Bass 1956, 1963, Perkins 1982 p 96). The latter case may be realized, for example, inside waveguides during the transmission of TE and TM waves. Due to the fact that waveguides are nothing but two-dimensional phase-locked cavities with the third dimension *open*, one can investigate not only the *staircase dynamics* of trapped radiation (Jennison and Drinkwater 1977), but also one-dimensional kinematic phenomena of the energy–momentum transfer mediated by longitudinal photons.

Let us consider a rectangular vacuum waveguide with perfectly conducting walls and a charge-free inside. If we assume that the direction of wave transmission coincides with the z-axis, the Maxwell and the Helmholtz equations read (Jackson 1975)

$$\partial_\mu \partial^\mu \phi_{\eta\xi}(x,y) \exp[kz - \omega t] = 0 \tag{22a}$$

$$\kappa^2_{\eta\xi} = \omega^2 c^{-2} - k^2 \tag{22b}$$

$$(\Delta + \kappa^2_{\eta\xi})\phi_{\eta\xi}(x,y) = 0 \tag{22c}$$

$$\phi_{\eta\xi} = \cos\left(\frac{\eta\pi x}{a}\right)\cos\left(\frac{\xi\pi y}{b}\right) \qquad \kappa^2 = \pi^2\left(\frac{\eta^2}{a^2}+\frac{\xi^2}{b^2}\right) \qquad \eta,\xi = 0,1,2,3\ldots \tag{23}$$

where a, b are transverse dimensions of the waveguide. The transition from the scalar solutions of (22*a*) to the vector one, may be accomplished by a simple transformations

$$\boldsymbol{E} = \nabla\times(\phi\boldsymbol{n}_z) \qquad \boldsymbol{H} = \nabla\times\nabla\times(\phi\boldsymbol{n}_z) \qquad \phi = \phi_{\eta\xi}(x,y)\exp[\mathrm{i}(kz-\omega t)]. \tag{24}$$

Defining the so-called *cutoff frequency*, $\omega_{\eta\xi} = c\kappa_{\eta\xi}$, the group and phase velocities of propagation may be given (Jackson 1975) as follows

$$V_{\mathrm{G}} = \mathrm{d}\omega/\mathrm{d}k = c[1-(\omega_{\eta\xi}/\omega)^2]^{1/2} \leqslant c \qquad V_{\mathrm{P}} = \omega/k = c[1-(\omega_{\eta\xi}/\omega)^2]^{-1/2} \geqslant c. \tag{25}$$

On the other hand, the power and the energy transferred down the guide are described by the wellknown formulae (Jackson 1975)

$$\begin{aligned} W &= \frac{c}{2}\left(\frac{\omega}{\omega_{\eta\xi}}\right)^2\left(1-\frac{\omega_{\eta\xi}^2}{\omega^2}\right)^{1/2}\int_{0,0}^{a,b}\phi^*\phi\,\mathrm{d}x\,\mathrm{d}y \\ E &= \frac{1}{2}\left(\frac{\omega}{\omega_{\eta\xi}}\right)^2\int_{0,0}^{a,b}\phi^*\phi\,\mathrm{d}x\,\mathrm{d}y. \end{aligned} \tag{26}$$

A detailed analysis of (25) and (26) leads to interesting conclusions:

(i) phase and group velocities of the transferred radiation satisfy the relation $V_{\mathrm{P}}V_{\mathrm{G}}/c^2 = 1$ identical to that for matter waves connected with a particle moving at the velocity V_{G},

(ii) a power-to-energy ratio gives the same result as that for momentum-to-energy ratio in relativistic mechanics: $W/E = kc^2/\omega = pc^2/E = V_{\mathrm{G}} \leqslant c$, suggesting a *corpuscular* character of the transferred radiation.

Now, if we focus our attention on (22*c*), it becames transparent that the *cutoff frequency* may be treated as a Compton frequency of longitudinal photons endowed with a rest mass $m_{\eta\xi}^0 = \hbar\omega_{\eta\xi}c^{-2}$. Such photons have the bradyon type characteristics

$$\omega^2 - k^2c^2 = \omega_{\eta\xi}^2 \qquad E^2 - p^2c^2 = (m_{\eta\xi}^0 c^2)^2 \tag{27a,b}$$

$$\begin{aligned} m_{\eta\xi} &= m_{\eta\xi}^0[1-(V_{\mathrm{G}}/c)^2]^{-1/2} \\ E &= m_{\eta\xi}c^2 \\ p &= m_{\eta\xi}V_{\mathrm{G}} = m_{\eta\xi}^0 V_{\mathrm{G}}[1-(V_{\mathrm{G}}/c)^2]^{-1/2} \end{aligned} \tag{28}$$

and the same relativistic properties as the *ordinary* ponderable matter, for instance;

(i) they cannot move at the light velocity because this case holds for $\omega = \infty$;

(ii) for $\omega = \omega_{\eta\xi}$, photons are at rest $V_{\mathrm{G}} = 0$, $V_{\mathrm{P}} = \infty$;

(iii) in the case of the luminal TEM waves $\Delta\phi(x,y)=0$, $\kappa=0$, $V_G=V_P=c$, and the associated (transverse) photons are exactly massless.

These surprising conclusions become comprehensible in the context of the previous section. Namely, the solutions of the Helmholtz equation ($\kappa \neq 0$) describe the standing waves measured normal to the planes xz and yz, and the mode numbers (η,ξ) determine the number of half wavelenghts between the conducting walls. The application of the boundary conditions, leads to the appearance of the standing wave modes, strictly related to the quanization of associated mass

$$m^0_{\eta\xi} = \hbar\pi c^{-1}\left(\frac{\eta^2}{a^2}+\frac{\xi^2}{b^2}\right)^{1/2}. \tag{29}$$

Such trapped radiation has inertial properties of a particle, with the possibility of the *excitation* of different inertial states; on the other hand, intrinsically stable massive objects may be considered as the modes of imprisoned radiation. These conclusions are in full agreement with the result of Jennison and Drinkwater (1977) and Jennison (1978) who have shown that the acquisition of different inertial states is quantized. In passing one may note that the outlined concept is consistent with the de Broglie (1951) theory of *constrained* particle states. According to de Broglie, the *constrained* states present an illuminating analogy to the circumstances one encounters for photons enclosed in a waveguide, whose motions correspond to rest masses which vary according to the form of the waveguide and the type of wave propagation. Such massive photons were suggested to have rest masses much greater than the normal rest mass of the photon, which is zero or undetectably small (de Broglie 1951).

6. Proca wave equation for longitudinal photons

The preceding chapter has been devoted to the *corpuscular* mechanics of longitudinal photons which are wave objects in the strict sense of these words. In view of the above, it is tempting to try to find the relativistic field equation describing propagation of matter waves associated with the time-like component of massive photons. Because the wave–particle duality is the genuine property of the system under consideration, it would be interesting to derive such equation without using the standard quantization formalism. The expression searched for should be identical to that obtained by the application of the quantization law $E \to \hat{E}$, $p \to \hat{p}$ to (27*b*). In order to accomplish this we start with the general equation of propagation given as

$$\left(\frac{1}{V_P^2}\frac{\partial^2}{\partial t^2} - \Delta\right)\psi(t,x,y,z) = 0 \tag{30}$$

which for photons moving along the z-axis reduces to the form

$$\left(\frac{1}{V_P^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2}\right)\psi(t,z) = 0 \tag{31a}$$

$$\psi(t,z) = \exp[i(kz-\omega t)] \tag{31b}$$

$$V_P = c[1-(\omega_{\eta\xi}/\omega)^2]^{-1/2}. \tag{31c}$$

Substitution of (31*c*) into (31*a*), and then differentiation ψ twice with respect to the time coordinate yields

$$\left([1-(\omega_{\eta\xi}/\omega)^2]\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial z^2}\right)\exp[\mathrm{i}(kz-\omega t)]$$
$$=\left(\partial_\mu\partial^\mu+(m^0_{\eta\xi}c/\hbar)^2\right)\psi(t,z)=0 \tag{32}$$

which is the relativistic Proca wave equation (Proca 1930, 1931, 1936) for longitudinal massive photons of a rest mass $m^0_{\eta\xi}$, moving in the z direction. This expression is equivalent to the Klein–Gordon equation for a spin-zero free-moving particle, which in the standard approach has been obtained via the correspondence rule and the application of energy–momentum operators.

7. Tachyonic component of longitudinal photons

The massive photons considered in sections 5 and 6 are of bradyon type, e.g. they are slower-than-light objects ($V_\mathrm{G} < c$), endowed with the real rest mass and the time-like 4-momentum. On the other hand, the associated matter waves propagate at the superluminal phase velocity and the subluminal group one, since they have the B-wave characteristic. As already stated, according to the two-wave description of matter and the tachyonic theory of elementary particle structure, the particles may be treated as composite objects with both bradyonic and tachyonic components. The formation of a time-like component (connected with B-waves) should accompany the creation of a space-like component associated with D-waves. Bearing in mind relations (7), the D-waves are postulated to have the following characteristics:

$$V'_\mathrm{P}=\omega'/k'=c[1-(\omega_{\eta\xi}/\omega)^2]^{1/2}\qquad V'_\mathrm{G}=\mathrm{d}\omega'/\ dk'=c[1-(\omega_{\eta\xi}/\omega)^2]^{-1/2} \tag{33}$$

Proceeding along the line of the previous sections, and taking advantage of (4*a,b*) and (6*b*), we get

$$\omega'^2-k'^2c^2=-\omega^2_{\eta\xi}\qquad E'^2-p'^2c^2=-(m^0_{\eta\xi}c^2)^2 \tag{34}$$

$$m_{\eta\xi}=m'^0_{\eta\xi}[(V'_\mathrm{G}/c)^2-1]^{-1/2}$$
$$E'=m'_{\eta\xi}c^2$$
$$p'=m'_{\eta\xi}V'_\mathrm{G}=m^0_{\eta\xi}V'_\mathrm{G}[(V'_\mathrm{G}/c)^2-1]^{-1/2}$$
$$V'_\mathrm{P}=c[1+(\omega_{\eta\xi}/\omega')^2]^{-1/2}\qquad V'_\mathrm{G}=c[1+(\omega_{\eta\xi}/\omega')^2]^{1/2}. \tag{35a,b}$$

The derived relativistic equations, describe the tachyonic component of massive photons, endowed with the space-like 4-momentum and the superluminal 3-velocity. Expressions (35*a*) and (35*b*) are equivalent to (33) and represent the phase and group velocities of the associated D-wave. In order to obtain the equation governing the propagation of the D-wave, we proceed analogously as in section 6, starting with the general wave formula

$$\left(\frac{1}{V'^2_\mathrm{P}}\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial z^2}\right)\psi'(t,z)=0 \tag{36a}$$

$$\psi'(t,z)=\exp[\mathrm{i}(k'z-\omega' t)] \tag{36b}$$

$$V'_\mathrm{P}=c[1+(\omega_{\eta\xi}/\omega')^2]^{-1/2}. \tag{36c}$$

Putting (36c) into (36a), and then by differentiating ψ' twice with respect to the time coordinate one gets the space-like counterpart of the Proca wave equation

$$\left([1+(\omega_{\eta\xi}/\omega')^2]\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial z^2}\right)\exp[\mathrm{i}(k'z-\omega' t)]$$
$$=\left(\partial_\mu\partial^\mu-(m^0_{\eta\xi}c/\hbar)^2\right)\psi'(t,z)=0. \tag{37}$$

This equation, unknown in electrodynamics, describes the propagation of a tachyonic field associated with the space-like component of longitudinal photons, and is equivalent to the Feinberg (1967) wave equation for faster-than-light objects.

8. Extended electromagnetism

The problem of extending electromagnetism to space-like objects is not straightforward. The fundamental requirement of such a generalized theory is the invariance of the electromagnetic tensor $F^{\mu\nu}$ and the 4-potential A^μ under superluminal Lorentz transformations. Such operations change a time-like tangent vector into a space-like one, and vice versa (i.e. invert the quadratic-form sign), and form a new extended group **G** together with the subluminal (orto- and anti-chronous) Lorentz transformations (Recami 1986 p54). However, the extended relativistic theories, including the superluminal inertial frames, encounter some interpretative difficulties unless they are formulated in the pseudo-Euclidean space-times $M(n,n)$ having the same number n of space and time dimensions (Recami 1986 pp 39, 118). In view of the above only the two-dimensional representation of the **G**-group, acting in the $M(1,1)$ space, has clear physical interpretation (Maccarrone and Recami 1982).

If one assumes $F^{\mu\nu}$, A^μ to be **G**-invariant

$$T^{\mu\nu}=G^\mu_\alpha G^\nu_\beta T'^{\alpha\beta} \qquad A^\nu=G^\nu_\alpha A'^{\alpha} \tag{38}$$

then ordinary Maxwell equations keep their form also for space-like objects (Recami and Mignani 1974, p277). In view of the above, we can write down the basic equations for the extended electromagnetism, including the space-like components of massive photons as follows

$$(\partial_\mu\partial^\mu-m^2)A_\alpha=j_\alpha \qquad j^\alpha=\rho_0 v'^{\alpha} \qquad \partial_\alpha j^\alpha=0 \tag{39}$$

$$\partial^\mu F_{\mu\alpha}-m^2A_\alpha=j_\alpha \qquad F_{\mu\alpha}=\partial_\mu A_\alpha-\partial_\alpha A_\mu \tag{40a,b}$$

where j^α, ρ_0 are the 4-current and the density of electric charges. Expression (40a) is the space-like counterpart of the famous Proca wave equation for a massive vector field coupled to a conserved current. In the 3-vector notation the *massive* Maxwell equations corresponding to (39)–(40) become

$$\nabla\cdot\boldsymbol{E}=\rho_0+m_0^2A_0 \qquad \nabla\times\boldsymbol{E}=-\frac{1}{c}\frac{\partial\boldsymbol{H}}{\partial t} \tag{41}$$

$$\nabla\cdot\boldsymbol{H}=0 \qquad \nabla\times\boldsymbol{H}=\boldsymbol{j}+m_0^2\boldsymbol{A}. \tag{42}$$

Adopting the results of de Broglie (1957) and Bass and Schrödinger (1955), one can write the energy-momentum density of free electromagnetic fields associated with the space-like component of a massive photon

$$E = \tfrac{1}{2}[\boldsymbol{E}^2 + \boldsymbol{H}^2 - m_0^2(\boldsymbol{A}^2 + A_0^2)] \qquad \boldsymbol{p} = \frac{1}{c}[\boldsymbol{E} \times \boldsymbol{H} - m_0^2 A_0 \boldsymbol{A}] \tag{43}$$

which imply the conservation of the continuity equation

$$\frac{1}{c}\frac{\partial E}{\partial t} + \nabla \cdot \boldsymbol{p} = 0. \tag{44}$$

For space-like objects, equations (43) have a physical meaning only for

$$\boldsymbol{E}^2 + \boldsymbol{H}^2 \geqslant m_0^2(\boldsymbol{A}^2 + A_0^2) \qquad \boldsymbol{E} \times \boldsymbol{H} \neq m_0^2 A_0 \boldsymbol{A} \tag{45}$$

and the case $\boldsymbol{E}^2 + \boldsymbol{H}^2 = m_0^2(\boldsymbol{A}^2 + A_0^2)$ corresponds to the infinite speed of the group velocity, allowed for tachyons. Before closing this section it may be noted that Lorentz gauge $A^\mu \to A'^\mu + \partial_\nu \Phi$, for massive vector fields, admits only a constant value of a scalar function Φ. It is easy to verify, because from the Lorentz gauge $\partial_\mu A^\mu = 0$ we get $\partial_\mu \partial^\mu \Phi = 0$, on the other hand, the Proca wave equation leads to the second requirement $\partial_\alpha(\partial_\mu \partial^\mu - m^2)\Phi = 0$. In consequence, $\partial_\alpha \Phi = 0$, and all freedom of gauge transformation for the time-like as well as the space-like massive photons is lost.

9. Conclusions

The hypothetical space-like objects and the space-like particle states, even if speculative, deserve some attention as they may play a role in the elementary particle structure (Hamamoto 1972, Mignani and Recami 1975, Guenin 1976, Rafanelli 1974, 1976, 1978, Recami 1986), as well as in the field theories of matter (Taylor 1976, Nielsen and Olesen 1978). The incorporation of tachyons in physics permits better understanding of many aspects of the *ordinary* relativistic theories, and the reproduction of results at both classical and quantum levels (Recami 1986). In this paper, an attempt was made to include faster-than-light objects in the electromagnetism, by an extension of the time-like Proca theory of massive photons. The obtained space-like counterpart of the Proca wave equation describes propagation of a tachyonic vector field associated with the space-like component of the longitudinal photons, and may be exploited in the investigation of photon conversion into ponderable matter, and vice versa.

It is well-known that transverse photons as carriers of vector field with spin $J = 1$, can exist only in two states with the spin projection on the direction of motion $J_{\mathrm{p}} = \pm 1$ corresponding to the helicity $\lambda = \pm 1$. However, the massive photons (for example: *virtual* photons carrying static interactions between charges) have the third degree of freedom, connected with longitudinal polarization and spin projection $J_p = 0$. The tachyonic field associated with such photons may be interpreted as the space-like component of a *virtual* photon (or Higgs-like particle), whose 4-momentum is orthogonal to the 4-momentum of the *ordinary* bradyonic component.

It is interesting to note that the double wave–particle characteristics given by equations (*6a,b*) are of photon type (*11a*). This suggests an internal photon-like structure of particles (Horodecki 1988a) and the photon origin of ponderable matter, both in full agreement with the results of this work. The idea that mass has an electromagnetic origin was developed long ago (Hasenöhrl 1904, 1905, Sedlak 1986, Winterberg 1987) and has played a fundamental role in many cosmological models of the universe. For example, in the Klein–Alfven and the Charon models, as well as in the standard model, it has been assumed that all particles have originated from photons of high energy at the beginning of the cosmic evolution. This suggests that extended electromagnetism, including the space-like objects and the suitable cosmological metric, would be the best framework for the description of particles creation in the early stages of the universe.

Finally, let us recall an interesting suggestion (Jennison 1978) that the internal nodes of the standing electromagnetic waves trapped into a cavity may be considered as a quark model. In view of the above, it is tempting to try to generalize this concept even to the dual-resonance models which conceive particles as non-local objects, e.g. strings. Such authors as Tze (1974) and Barut (1978) underlined the connections between electromagnetic and dual strings; note, moreover, that the investigation of the string models at both the classical and quantum levels has predicted the presence of tachyons in the spectrum of states.

The electromagnetic approach to the elementary particle structure, was also presented by Jehle (1971, 1972), Post (1983, 1986) and Elbaz (1987, 1988). In particular, Jehle accomplished a consistent formulation of leptons, quarks and hadrons, in terms of electromagnetic fields and their amplitude distributions; he has shown that the topological structure of those fields represents the internal quantum numbers in the particle physics.

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